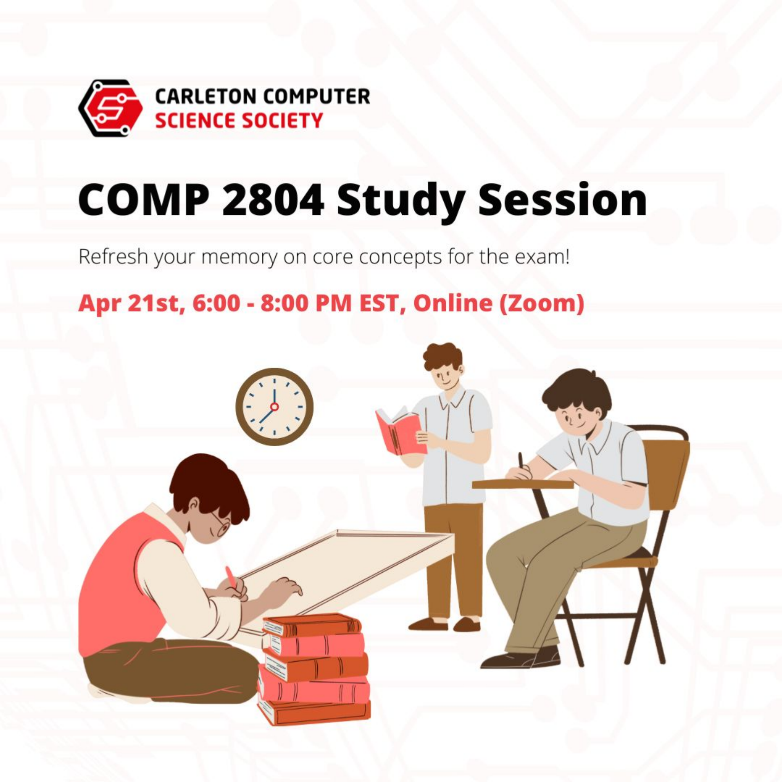

COMP 2804 Study Session - Winter 2025

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Study Session Format:

- Go over possible topics on the exam
- List tips and tricks to succeed (based on intuition and past experience)
- Go over 10 practice questions across all 5 chapters
  - Counting
  - Recursion
  - Discrete Probability
  - Random Variables and Expectation
  - Probabilistic Method
- Open Q & A - will take questions from chat and attempt to answer them

DISCLAIMER:

- The material presented in this slides and during the study session will not influence what is on the final exam in one way or another - this is not the definitive content of the final.
- This is not a comprehensive study guide - it is more intended to complement your studies.
- Please review the content from the professor and the textbook as well.

Tips and Tricks For Exam:

1. No cheat sheet, so memorize common formulas (ex. Linearity of Expectation, Independent Events)

2. Test on small examples (especially useful if the question is not computational or the question is recursive)

3. Study past assignments over past exams (better preparation for "show your work" style questions)
   a. I would be careful with textbook questions since there aren't any known accepted solutions

Sample/Practice Questions

- 9 Questions
  - 1 Counting Question
  - 2 Recursion Question
  - 3 Probability Questions
  - 3 Random Variables and Expectation Questions

Question 1

A *flip* in a bitstring is a pair of adjacent bits that are not equal. For example, the bitstring `010011` has three flips: The first two bits form a flip, the second and third bits form a flip, and the fourth and fifth bits form a flip.

- Determine the number of bitstrings of length 7 that have exactly 3 flips at the following positions: The second and third bits form a flip, the third and fourth bits form a flip, and the fifth and sixth bits form a flip.
- Let $n \geq 2$ and k be integers with $0 \leq k \leq n-1$. Determine the number of bitstrings of length n that have exactly k flips.

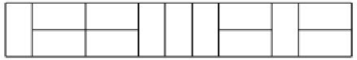
Question 2

4.24 Let $n \geq 1$ be an integer and consider a $2 \times n$ board B_n consisting of $2n$ square cells. The top part of the figure below shows B_{13}.

A *brick* is a horizontal or vertical board consisting of 2 square cells; see the bottom part of the figure above. A *tiling* of the board B_n is a placement of bricks on the board such that

- the bricks exactly cover B_n and
- no two bricks overlap.

The figure below shows a tiling of B_{13}.



For $n \geq 1$, let T_n be the number of different tilings of the board B_n. Determine the value of T_n, i.e., express T_n in terms of numbers that we have seen in this chapter.

Question 3

Algorithm DigDirt(n):

if $n = 1$:

Drink water for **one minute**;

return;

else

Shovel for n minutes;

DigDirt($\frac{n}{3}$);

DigDirt($\frac{n}{3}$);

You should assume that $n = 3^m$ for an integer m (or that n is always a multiple of 3).

Determine the number of minutes you drink water as a function of m.

Question 4

A standard deck consists of a set of 52 cards. There are 13 distinct ranks,

$$\{A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K\}.$$

There are 4 cards of each rank, one of each of the following 4 suits: $\{\heartsuit, \diamondsuit, \spadesuit, \clubsuit\}$. In traditional poker, each player is dealt 5 cards initially. Assume the dealer is using a single deck of 52 cards and deals you a hand of 5 cards uniformly at random.

Define the events:

- F = You are dealt 5 cards of the same suit (called a *flush*).
- D = You are dealt at least one diamond.

For this question, F includes all possible hands of 5 cards with the same suit.

1. What is $\Pr(F)$?
2. What is $\Pr(D)$?
3. What is the probability of getting a flush given that there is at least one diamond in your hand?

Question 5

In the *5-107 Lottery* you choose a set of 6 distinct integers $\{x_1, \ldots, x_5, y\}$ from the set $\{1, 2, 3, \ldots, 107\}$. $x_1, \ldots, x_5$ are called your *main numbers* and y is your *bonus number*. On Friday night, the lottery machine draws a uniformly random 5-number subset $\{z_1, \ldots, z_5\}$ from $\{1, \ldots, 107\}$. You buy one lottery ticket with your favourite 6 numbers.

1. You win a **Big Jackpot** if $\{x_1, \ldots, x_5\} = \{z_1, \ldots, z_5\}$. What is the probability that you win the Big Jackpot?

2. You win a **Little Jackpot** if $|\{x_1, \ldots, x_5\} \cap \{z_1, \ldots, z_5\}| = 4$. What is the probability that you win a Little Jackpot?

Question 6

In Blackjack you want a hand that totals as close to 21 as possible without going over. The dealer will start by dealing you two cards, then deal themselves one card, all face up. For this question, we will assume that the dealer is using a standard deck of 52 cards. All numbered cards have a value equal to their number. All face cards (King = K, Queen = Q, Jack = J) are worth 10. Aces = A are worth 1 or 11.

If you are dealt an Ace and a King as your first two cards (in either order), you have Blackjack and will win if the dealer does not also get Blackjack.

Let D be the event that you've been dealt a Blackjack. That is, D is the event that you are dealt one Ace and one of $10, J, Q$, or K of any suit and in either order.

Let E be the event that the dealer deals themselves an Ace. That is, E is the event that the dealer's first card is in the set $\{A\heartsuit, A\diamondsuit, A\clubsuit, A\spadesuit\}$.

1. What is $\Pr(D \cap E)$?
2. What is $\Pr(D \cup E)$?
3. Are the events D and E independent?

Question 7

You go fishing in a lake. There are two types of fish (trout and pike) in the lake. Each time you cast your line, you catch a trout with probability 4/5 and you catch a pike with probability 1/5. You throw every fish you catch back in the water, so the result of each cast is independent of which types of fish you caught on previous casts. You stop fishing once you've caught at least one trout and at least one pike.

What is the expected number of casts you make before you stop fishing?

Question 8

Each month you are sent one of n different brands of pizza uniformly at random, and independently of previous orders.

Define the random variable X to be the total number of distinct brands of pizza that you receive. Determine the expected value $\mathbb{E}(X)$ of X.

Question 9

Let D be a standard deck of cards.

For a card $c \in D$, let the value of $X(c)$ be equal to the value of the card. So 1 if the rank is A, 2 if the card is 2, ..., and 10 if the rank is in $\{J, Q, K\}$.

Assume you are dealt a standard hand of 5 cards.

Let the folllowing be random variables:

$$V = \text{The sum of the values of the cards in your hand,}$$
$$F = \text{The number of face cards (that is, } J, Q, \text{ or } K\text{) in your hand.}$$

1. What are $\mathbb{E}(V)$ and $\mathbb{E}(F)$?
2. Are V and F independent random variables?

Resources:

Past assignments:

- https://cglab.ca/~michiel/2804/oldassignments/oldassignments.html

Past Midterms and Exams:

- https://cglab.ca/~michiel/2804/oldmidterms/oldmidterms.html
- https://cglab.ca/~michiel/2804/oldexams/oldexams.html
- https://cglab.ca/~morin/teaching/2804/oldexams.html

Interactive Version of Midterms and Exams:

- https://questions.carletoncomputerscience.ca/comp2804