Calculus Review Fall 2023

1. Find the limit of $\lim_{x\to 5} \dfrac{x+5}{x+4}$.
2. Find the limit of $\lim_{x\to 0} cos(\dfrac{\pi}{\sqrt{14-5cos2x}})$.
3. Find the limit of $\lim_{x\to 4} \dfrac{\frac{1}{x}-\frac{1}{4}}{x-4}$.
4. Find the limit of $\lim_{x\to -5^-} f(x)$ and $\lim_{x\to -5^+} f(x)$ given the function $f(x) = (x+11)\dfrac{|x+5|}{x+5}$.
5. Find the limit of $\lim_{\theta\to 0} \dfrac{\theta csc(8\theta)}{cos(10\theta)}$ using $\lim_{\theta\to 0} \dfrac{sin\theta}{\theta} = 1$.
6. Find the limit of $\lim_{x\to\infty} (\sqrt{36x^2+6x} - \sqrt{36x^2-5})$
7. Where is $y = log(8x-24)$ continuous?
8. Define $f(7)$ that extends $f(x) = \dfrac{8x^2-392}{8x-56}$ to be continuous at $x = 7$.
9. Find the equation of the tangent line of $y = 5-7x^2$ at $(3,-58)$
10. Find the equation of the tangent line of $f(x) = \dfrac{5x}{x-3}$ at $(4,20)$
11. Given $f(x) = 3x^2 - x + 2$, find the derivative using $\lim_{z\to x} \dfrac{f(z)-f(x)}{z-x}$
12. Given $f(x) = 3+\sqrt{5x}$, find the derivative using $\lim_{z\to x} \dfrac{f(z)-f(x)}{z-x}$
13. Find derivative of $f(x) = (2x^2-3x+8)(cos(x)+29^x)$
14. Find $\dfrac{dy}{dx}$ of $y = 6x^2 sin\ x + 12x cos\ x - 12 sin\ x$
15. Find $\dfrac{dy}{dx}$ of $y = \dfrac{6e^x}{7x}$
16. Find $\dfrac{dy}{dx}$ of $y = sin^2(9\pi x - 2)$
17. Differentiate $f(x) = ln[ln(ln(5x))]$
18. Find $\dfrac{dy}{dx}$ of $y = sin^{-1}(\sqrt{15}x)$
19. Implicitly differentiate $(7x^2+5)^2 = 28y$
20. Implicitly differentiate $e^{x^2y} = 7x + 6y + 3$
21. Use logarithmic differentiation to find the derivative of $y = \dfrac{x\sqrt{x^2+2}}{(x+4)^{5/3}}$
22. Find $f'(x)$ and $f"(x)$ of $f(x) = \dfrac{2x^5+6}{x^3}$
23. Implicitly differentiate to find the first and second derivative of $3x^2 + 2y^2 = 9$
24. Find the linearization $L(x)$ of $f(x) = cot\ x$ at $x = \dfrac{3\pi}{4}$

25. Find the absolute extrema of $f(x) = -4\sqrt{4-x^2}$, $-2 \leq x \leq 1$
26. Differentiate $\lim_{x\to 0} = \dfrac{3x^2}{2ln(sec\ x)}$ using l'Hôpital's Rule
27. Differentiate $\lim_{x\to 0} = \dfrac{\sqrt{3x+1}-1}{y}$ using l'Hôpital's Rule
28. Find limit of $\lim_{x\to\infty} (ln\ 4x - ln(x+6))$ using l'Hôpital's Rule
29. Find the limit of $\lim_{x\to 0^+} x^{-2/ln\ x}$

Solutions (Answers may not be correct)

1. $\dfrac{10}{9}$
2. $\frac{1}{2}$
3. $-\frac{1}{16}$
4. $\lim_{x\to -5^-} f(x) =$ -6, $\lim_{x\to -5^+} f(x) = 6$
5. $\frac{1}{8}$
6. $\frac{1}{2}$
7. $(3, \infty)$
8. Take $f(7) = 14$
9. $y = -42x + 68$
10. $80 - 15x$
11. $6x - 1$
12. $\dfrac{\sqrt{5}}{2\sqrt{x}}$
13. $f'(x) = (4x-3)(cos\ x + 29^x) + (-sin\ x + 29^x ln29)(2x^2 - 3x + 8)$
14. $6x^2 cos\ x$
15. $\dfrac{dy}{dx} = \dfrac{6e^x(x-1)}{7x^2}$
16. $18\pi cos(9\pi x - 2) sin(9\pi x - 2)$
17. $\dfrac{1}{xln(5x)ln(ln(5x))}$
18. $\dfrac{dy}{dx} = \dfrac{\sqrt{15}}{\sqrt{1-15x^2}}$
19. $\dfrac{dy}{dx} = 7x^3 + 5x$
20. $\dfrac{dy}{dx} = \dfrac{7 - 2e^{x^2 y} xy}{e^{x^2 y} x^2 - 6}$

21. $\frac{dy}{dx} = (\frac{1}{x} + \frac{x}{x^2+2} - \frac{5}{3(x+4)})(\frac{x\sqrt{x^2+2}}{\sqrt[3]{(x+4)^5}})$

22. $f'(x) = \frac{2x^5 - 9}{x^4}$, $f"(x) = \frac{4x^5 + 72}{x^5}$

23. $\frac{dy}{dx} = \frac{-3x}{2y}$, $\frac{d^2y}{dx^2} = \frac{-3(2y^2 + 3x^2)}{4y^3}$

24. $L(x) = -1 - 2(x - \frac{3\pi}{4})$

25. Maximum $(-2, 0)$, minimum $(0, -8)$

26. 3

27. $\frac{3}{2}$

28. $ln\ 4$

29. $\frac{1}{e^2}$