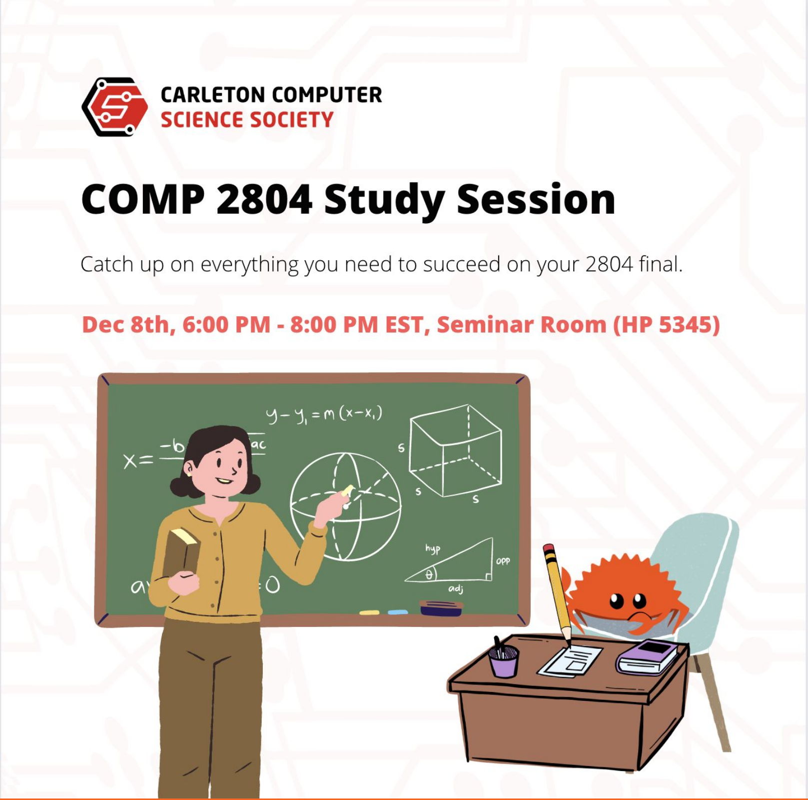

COMP 2804 Study Session

By: Nguyen-Hanh Nong

What is this for?

Simultaneous collaborative review session/tips and tricks for the exam

Problem:

Final Examination	40%

Topics in the Course

Counting
- Product Rule
- Bijection Rule
- Complement Rule
- Sum Rule
- Inclusion/Exclusion Principle
- Combinatorial Proofs*
- Pigeonhole Principle

Probability
- Probability Spaces
- Conditional Probability
- Law of Total Probability and Bayes Theorem
- Independent Events
- Infinite Probability Spaces

Recursion
- Recursive Functions
- Recurrence Relations

Random Variables and Expectation
- Random Variables
- Expected Values
- Linearity of Expectation
- Geometric and Binomial Distribution
- Indicator Random Variables
- Probabilistic Method

This Color = Rarer on exams/Haven't seen on a past exam yet

This Color = Definitely should study/very likely to appear on exam

Review of Winter 2017 Exam:

Tips and Tricks (probably obvious but worth reiterating)

Tip #1: Computational Real Number Questions First

- There's probably going to be questions on the exam that do not involve actual numbers (involve terms like n and i)
- Tendency for those questions to be more theoretical (asking you to understand applying certain theories and/or rule)
- Computational questions should generally be done first, if you're more comfortable with them

Example of non-computational question (Winter 2019 Exam)

5. Let $m \geq 2$ and $n \geq 2$ be integers. Why does the identity

$$\binom{m+n}{2} = \binom{m}{2} + \binom{n}{2} + mn$$

hold?

- ☐ Because both sides count the number of ways m men and n women can be arranged on a line, such that not two men are standing next to each other.
- ☐ Because both sides count the number of ordered pairs in a set of size $m+n$.
- ☐ Because both sides count the number of 2-element subsets of a set of size $m+n$.
- ☐ None of the above.

Example of computational question (Winter 2019 Exam)

4. Consider the sets $A = \{1, 2, \ldots, 10\}$ and $B = \{1, 2, \ldots, 14\}$. Let $S = \{(x, y) : x \in A, y \in B\}$. An element (x, y) of S is *awesome*, if x is even or y is even. What is the number of awesome elements in S?

- ☐ 104
- ☐ 105
- ☐ 106
- ☐ 107

Tip #2: Pattern Match as much as possible

- **Pattern Matching:** “act of checking a given sequence of tokens for the presence of the constituents of some pattern.”
- Basically, copying and pasting techniques for questions that look similar
- Usually works good (since the exams don’t switch up between years that much), but don’t depend on it for you to pass ¯_(ツ)_/¯
- Best topics/types of questions to 1-1 pattern match: Recursion/recurrence type questions and counting/bitstring questions
- Works especially well when you don’t really know what you’re doing (possibly - maybe - probably might occur once on the exam)

Example of optimal pattern matching

Question 9, Fall 2018 Exam

9. Consider bitstrings that do not contain 110. Let S_n be the number of such strings having length n. Which of the following is true for any $n \geq 4$?

- $S_n = S_{n-1} + S_{n-2} + 1$
- $S_n = S_{n-1} + S_{n-2} + 2^{n-2}$
- $S_n = S_{n-1} + S_{n-2} + 2^{n-3}$
- $S_n = S_{n-1} + S_{n-2} + S_{n-3}$

Question 14 (a). A string over the alphabet $\{a, b, c\}$ is called *super* if it does not contain *abc*, *aba*, or *aa*. For $n \geq 1$, let A_n denote the number of super strings of length n. Which of the following is true for any $n \geq 4$?

- $A_n = A_{n-1} + A_{n-2} + A_{n-3}$
- $A_n = 2A_{n-1} + A_{n-2} + A_{n-3}$
- $A_n = 2A_{n-1} + 2A_{n-2} + A_{n-3}$
- $A_n = 2A_{n-1} + 2A_{n-2} + 2A_{n-3}$
- None of the other answers is correct

Question 14, Fall Winter 2022 Midterm (Version A)

Tip #3: Test on Small Examples

- **Technique that is ideal for solving non-computational questions and recursive/probability problems**
- Downsides: Takes quite a bit of time, which you might not have on the exam
- Ideal: Test only up to like n >= 5 or 6, any more than that and usually your gonna have too many test cases **(remember that you probably won't have calculators)**
- Most of the time, you'll generally not solve the problem right away, but remove 1 or 2 of the possible answers from contention

Example of small-examples question (Winter 2014 Exam)

10. Consider the following recursive function:

$$\begin{aligned} f(0) &= 3, \\ f(n+1) &= f(n) + 10n + 2 \text{ for all integers } n \geq 0. \end{aligned}$$

Which of the following is true?

(a) for all $n \geq 0$: $f(n) = 5n^2 - 3n + 2$
(b) for all $n \geq 0$: $f(n) = 5n^2 - 3n + 3$
(c) for all $n \geq 0$: $f(n) = 5n^2 + 3n + 3$
(d) for all $n \geq 0$: $f(n) = 5n^2 - 2n + 3$

Test on b)

- f(0) = 3 -> correct
- f(1) = 5 -> correct
- f(2) = 17 -> correct

Test on d)

- f(0) = 3 -> correct
- f(1) = 6 -> this is wrong

Test on c)

- f(0) = 3 -> correct
- f(1) = 11 -> this is wrong

Test on a)

- f(0) = 2 -> this is wrong

Base case:

- f(0) = 3
- f(1) = f(0) + 10(0) + 2 = 5
- f(2) = f(1) + 10(1) + 2 = 17
- f(3) = f(2) + 10(2) + 2 = 39
- f(4) = f(3) + 10(3) + 2 = 71

Therefore, answer b) is correct by process of elimination

Good luck on your exams!

Resources:

Past assignments:

- https://cglab.ca/~michiel/2804/oldassignments/oldassignments.html

Past Midterms and Exams:

- https://cglab.ca/~michiel/2804/oldmidterms/oldmidterms.html
- https://cglab.ca/~michiel/2804/oldexams/oldexams.html
- https://cglab.ca/~morin/teaching/2804/oldexams.html

Interactive Version of Midterms and Exams (some of the questions won't load and/or might be incorrect):

- https://discretemath.ca/

Explanations + Solutions to Winter 2017 Exam (Unofficial):

- https://docs.google.com/document/d/12Lmtq5u58Cdn-R8WAo4y8Oowk6V2n72v-k8npGYDIfY/edit?usp=sharing